IMPLEMENTING PML BOUNDARY CONDITIONS IN TLM

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Abstract—**The numerical simulation of Maxwell's equations requires a discretization, which must be limited in space domain. An often used method is the implementation of absorbing boundary conditions (ABC). In the FDTD method the perfectly matched layer (PML) has proven to be an excellent type of ABCs in many applications. In this paper the PML is introduced and verified for the TLM method in the 3-dimensional case for orthogonal incidence.**

Theoretical Background

For the numerical simulation of electromagnetic field problems it is necessary to limit the simulated space. Especially for time domain methods, the computational resources required quickly exceed the memory and CPU speed of the computers available today. One way to implement absorbing boundary conditions is the Mur wall as described in [1]. Another type of absorbing boundary conditions were implemented by Liao in [2]. Both have been used successfully, but the damping achieved by these methods is not sufficient for many purposes. Thus, another method was originally introduced in the finite difference time domain (FDTD) method by Berenger [3], [4]. This new absorbing boundary condition is called perfectly matched layer (PML). The PML simulates an unphysical material which has, in the ideal case, a reflection coefficient $r = 0$ for all incident waves.

Among other interesting fields of application the transmission line matrix method (TLM) has proven to be a powerful tool for calculating electromagnetic fields [5]. The PML was already used in the two dimensional transmission line matrix method (2D-TLM) by Eswarappa and Hoefer in [6]. In this paper the PML is introduced in 3D-TLM for waves of orthogonal incidence.

For a PML a magnetic current density $\boldsymbol{J}_\mathrm{M}$ is introduced, so that Maxwell's equations for time harmonic processes can be written as

$$\begin{aligned} \mathbf{rot}\,\boldsymbol{E} &= -\boldsymbol{J}_\mathrm{M} - \mathrm{j}\,\omega\mu\boldsymbol{H} \qquad (1) \\ \mathbf{rot}\,\boldsymbol{H} &= \boldsymbol{J}_\mathrm{E} + \mathrm{j}\,\omega\epsilon\boldsymbol{E}. \end{aligned}$$

Inserting the material equations one obtains

$$\mathbf{rot}\,\boldsymbol{E} = -\left(\kappa_\mathrm{M} + \mathrm{j}\,\omega\mu\right)\boldsymbol{H} \qquad (2)$$

$$\mathbf{rot}\,\boldsymbol{H} = \left(\kappa_\mathrm{E} + \mathrm{j}\,\omega\epsilon\right)\boldsymbol{E}. \qquad (3)$$

In these equations κ_M and κ_E are the magnetic and electric conductivity respectively.

Finally, the equations (2) can be rewritten as

$$\Delta \boldsymbol{E} - (\kappa_\mathrm{M} + \mathrm{j}\,\omega\mu)\,(\kappa_\mathrm{E} + \mathrm{j}\,\omega\epsilon)\,\boldsymbol{E} = \mathbf{o}, \tag{4}$$

if the PML material is source free. In the PML layer the propagation constant is given by

$$\gamma_\mathrm{PML} = \sqrt{(\kappa_\mathrm{M} + \mathrm{j}\,\omega\mu)\,(\kappa_\mathrm{E} + \mathrm{j}\,\omega\epsilon)}. \tag{5}$$

To obtain a transmission of incident waves without any reflexion, the equation

$$\frac{\kappa_\mathrm{E}}{\epsilon} = \frac{\kappa_\mathrm{M}}{\mu} \tag{6}$$

must be valid. The complex propagation constant can then be calculated as

$$\underline{\gamma} = \sqrt{\mu\epsilon}\left(\frac{\kappa_\mathrm{E}}{\epsilon} + \mathrm{j}\,\omega\right) = \alpha + \mathrm{j}\,\beta. \tag{7}$$

It is important to notice that the total absence of reflexions is only possible for the continuous case, whereas in the discretized space, the reflexion is very small, but not zero.

In the FDTD method the PML is used for the simulation of various problems with great success. For FDTD the PML has been implemented for 3D layer structures with one dimension assumed periodical for an arbitrarily incident wave [7]. This is achieved by dividing up the incident wave into one part propagating orthogonally to the PML layer and another part propagating perpendicularly.

A similar approach for the TLM is only implemented for the two dimensional case [6] by mixing an FDTD and a TLM mesh. The computational cost of this implementation is very high. Thus, the PML has not yet been implemented for arbitrary angles of the incident wave for 3D-TLM simulation.

This work is limited to the orthogonal incidence, which can be used for coplanar waveguides and microstrip lines [8]. For the symmetrical condensed node (SCN) [9] the electric and magnetic conductivity must be introduced into the TLM network for the three dimensional case. The resistance R and the conductance G are given by

$$R = \frac{\kappa_\mathrm{E}\mu}{Z_\mathrm{TLM}\epsilon} \cdot \Delta\ell \tag{8}$$

and

$$G = \kappa_\mathrm{E} \cdot Z_\mathrm{TLM} \cdot \Delta\ell. \tag{9}$$

Here, R describes the magnetic losses and G represents the electric losses. By calculating the quotient of the above two equations, the discretization step $\Delta\ell$ can be eliminated

$$\frac{R}{G} = \frac{\mu_\mathrm{r}}{\epsilon_\mathrm{r}}. \tag{10}$$

On the outside the PML is limited by an electric wall, by which waves are reflected back into the structure. Thus, the thickness of the PML must be at least several nodes, where the electric conductivity increases from zero to a certain value κ_max on the outside end of the PML.

In this work the electric conductivity inside the PML layer is described by functions of the type

$$p_n(x) = \kappa_\mathrm{max} \cdot \left(\frac{x}{D}\right)^n, \tag{11}$$

where D is the thickness of the PML layer and the layer is assumed to begin at $x = 0$. The reflexion characteristics of the PML layer can be controlled both by the thickness D of the layer and by the order n, which is the exponent in (11). Both parameters must be chosen carefully to obtain optimum performance of the PML.

Results and Discussion

As an example to prove the applicability of the PML for TLM simulation the effective permittivity of a coplanar waveguide is calculated numerically. Since the numerical calculation of the effective permittivity is significantly influenced by the characteristics of the absorbing boundaries, it is well suitable to underline the excellent performance of the PML ABC in TLM.

The TLM algorithm used in this paper is based upon an optimization of the scattering matrix as explained in [10]. To compute the effective permittivity, voltages and currents are defined at two transverse planes of the waveguide with distance ℓ. Between the two planes a phase shift in the currents and voltages can be calculated. Thus, the effective permittivity can be written as

$$\epsilon_{\mathrm{r,eff}} = \left(\frac{c_0 \cdot \varphi(\omega)}{\omega \ell} \right)^2 , \qquad (12)$$

where $\varphi(\omega)$ is the phase shift either of the voltages or of the currents. For non-ideal absorbing boundaries, the voltage phase shift and the current phase shift yield different effective permittivities due to reflections of the boundaries. The average between the two calculations can be used as an approximation for the effective permittivity.

The coplanar waveguide used in this paper has a slotwidth of 50 μm, an inner conductor width of 75 μm and a substrate layer thickness of 100 μm. The metalization thickness (3 μm) is approximated by a single metalization layer ($t \rightarrow 0$ μm), as shown in [11]. Mur absorbing boundaries of first order are used, and the effective permittivity is calculated by both voltage and current phase shift.

The results are compared to a similar TLM simulation, but with PML absorbing boundaries used instead. The results are shown in Fig. 1. The average between the two graphs

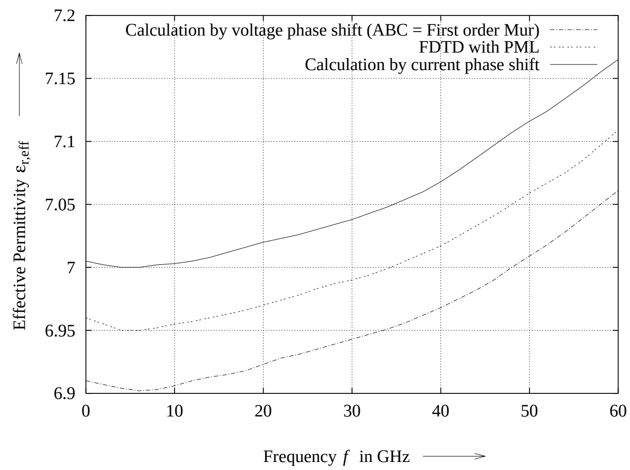

Fig. 1. Effective permittivity of a coplanar waveguide as function of frequency. The simulation of a first order Mur ABC is compared to a simulation using PML.

for the first order Mur ABC lies very close to the result obtained by using perfectly matched layer ABCs. Thus, the simulation with PMLs shows excellent agreement with the result of the simulation with Mur ABCs and it is no longer necessary to calculate the effective permittivity by the average of two computations.

In Fig. 2 the result for the simulation with PMLs is compared to an identical simulation made with the FDTD method using superabsorption boundary conditions. The two curves lie very close together for the whole frequency range from 0 to 60 GHz. Thus, both results show excellent agreement.

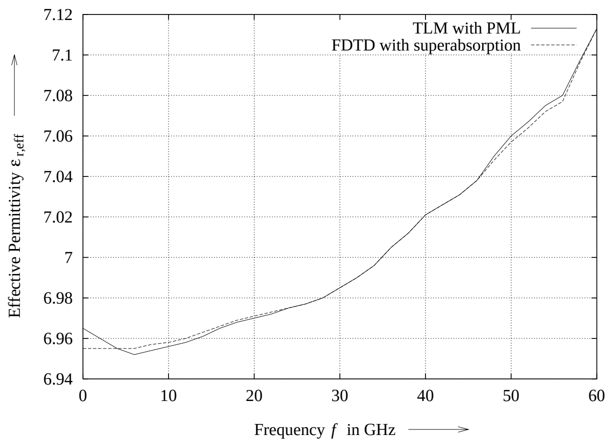

Fig. 2. Effective permittivity of a coplanar waveguide as function of frequency simulated with PML in TLM method and superabsorption boundary conditions in FDTD method respectively.

Conclusion

As conclusion it can be stated that the perfectly matched layer absorbing boundary condition has successfully been implemented in TLM for the three dimensional case, although still limited to orthogonal incidence of the electromagnetic wave.

Both the examples mentioned above and those planned for the presentation show the good applicability of the method developed here. For future time the method can hopefully be extended towards full threedimensional simulation with arbitrary angle of incidence.

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